team documented their *basis of assessment* (BOA) in Table 4.11. In Table 4.11, the ordinal scale level values shown derive from Tables 4.4 through 4.7.

Assessing Importance Weights and Computing the Overall Impact Score

The above discussion provides inputs to apply Equation 4.5 as one way to compute an overall impact score of risk event A. Before this can be done, importance weights for the evaluation criteria cost, schedule, technical performance, and programmatics must be determined.

From Equation 4.5, the overall impact score of risk event A is

$$V_{Impact}(A) = w_1 V_{Cost}(x_1) + w_2 V_{Sched}(x_2) + w_3 V_{TPerf}(x_3) + w_4 V_{Prgm}(x_4)$$

where w_i for $i = 1, \ldots, 4$ are non-negative weights (importance weights) for the cost, schedule, technical performance, and programmatics criteria. Recall these weights sum to one; that is,

$$w_1 + w_2 + w_3 + w_4 = 1$$

For this case, suppose the engineering team made the following importance weight assessments. Technical performance w_3 is twice as important as cost w_1; cost w_1 is twice as important as schedule w_2; cost w_1 is twice as important as programmatics w_4. From this, we have the following:

$$w_3 = 2w_1; \quad w_1 = 2w_2; \quad w_1 = 2w_4$$

Since $w_1 + w_2 + w_3 + w_4 = 1$ it follows, from the above relationships, that

$$w_1 + \frac{1}{2}w_1 + 2w_1 + \frac{1}{2}w_1 = 1$$

thus, $w_1 = \frac{1}{4}$. From this, $w_2 = \frac{1}{8}$, $w_3 = \frac{1}{2}$, and $w_4 = \frac{1}{8}$. Substituting these values into Equation 4.5 we have, for this case,

$$V_{Impact}(A) = \frac{1}{4}V_{Cost}(x_1) + \frac{1}{8}V_{Sched}(x_2) + \frac{1}{2}V_{TPerf}(x_3) + \frac{1}{8}V_{Prgm}(x_4) \qquad (4.6)$$

In Table 4.11, we have the values for the other terms in Equation 4.6; specifically,

$$V_{Impact}(A) = \frac{1}{4}(0.842) + \frac{1}{8}(0.604) + \frac{1}{2}(0.60) + \frac{1}{8}(0.79) = 0.685 \qquad (4.7)$$

TABLE 4.11: Illustrative Impact (Consequence) Assessments and Scores

Impact Assessments	Basis of Assessment (BOA)
Ordinal Scale Level (Score)	Risk Statement *Inadequate synchronization of the communication system's new database with the existing subsystem databases, because the new database for the communication system will not be fully tested for compatibility with the existing subsystem databases when version 1.0 of the data management architecture is released.* The consequent event descriptions in Figure 4.14 provide a starting point for the basis of assessments below. They support the team's judgments and supporting arguments for articulating the risk event's consequences, if it occurs, on the project's cost, schedule, programmatics, and the system's technical performance.
Cost Impact Level 4	This risk event, if it occurs, is estimated by the engineering team to cause a 12 percent increase in the project's current budget. The estimate is based on a careful assessment of the ripple effects across the project's cost categories for interoperability-related fixes to the databases, the supporting software, and the extent that re-testing is needed. Value Function Value: From Equation 4.2, $V_X(12) = 1.096(1 - e^{-12/8.2}) = 0.842$
Schedule Impact Level 3	This risk event, if it occurs, is estimated by the engineering team to cause a 4 month increase in the project's current schedule. The estimate is based on a careful assessment of the ripple effects across the project's integrated master schedule for interoperability-related fixes to the databases, the supporting software, and the extent that re-testing is needed.

TABLE 4.11: Illustrative Impact (Consequence) Assessments and Scores (*Continued*)

Impact Assessments	Basis of Assessment (BOA)
	Value Function Value: From Equation 4.3, $V_X(4) = 1.018(1 - e^{-4/4.44}) = 0.604$
Technical Performance Impact Level 4	This risk event, if it occurs, is assessed by the engineering team as one that will impact the system's operational capabilities to the extent that technical performance is marginally below minimum acceptable levels, depending on the location and extent of interoperability shortfalls.
	Value Function Value: From Figure 4.12, $V_X(4) = 9/15 = 0.60$
Programmatic Impact Level 4	This risk event, if it occurs, is assessed by the engineering team as one that will impact programmatic efforts to the extent that one or more stated objectives for technical or programmatic work products (e.g., various specifications or activities) is marginally below minimum acceptable levels.
	Value Function Value: From Figure 4.12, $V_X(4) = 15/19 = 0.79$

Figure 4.15 shows a plot of this risk event — one with an assessed occurrence probability of 0.95 and an overall impact (consequence) score of 0.685 (from Equation 4.7). Overall, this might be considered a risk of a high-moderately concern. This concludes Case Discussion 4.1.

The analysis approach presented in Case Discussion 4.1 can be extended to multiple risk events. This is illustrated in Figure 4.16. Shown is a scatter plot of 25 risk events. Here, each event is analyzed by the same process just discussed. Each risk event is then plotted by its occurrence probability and its overall impact (consequence) to the project.

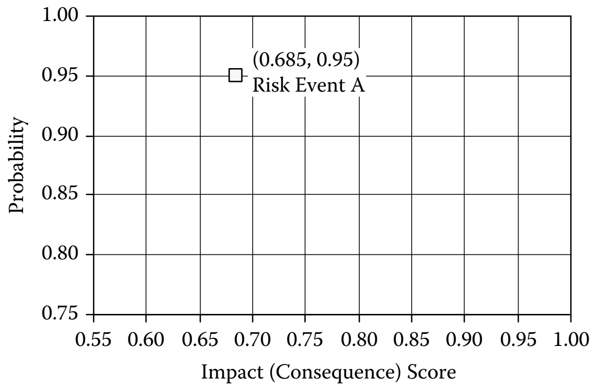

Figure 4.15: Case Discussion 4.1: a plot of risk event A.

In Figure 4.16, risk events 3, 5, and 8 appear to be ahead of the others in terms of occurrence probability and impact (consequence) to the project. What about risk events 24, 4, and 1? How important are they relative to risk events 3, 5, and 8? The following discusses an approach based on the preceding discussion for assessing the *relative rank-order* of risk events, when these events are presented in terms of a cardinal scatter plot.

An Algorithm for Ranking Risk Events

One way to develop a relative rank-order of risk events from information in a cardinal scatter plot is to apply the formulation shown in Equation 4.8. Equation 4.8

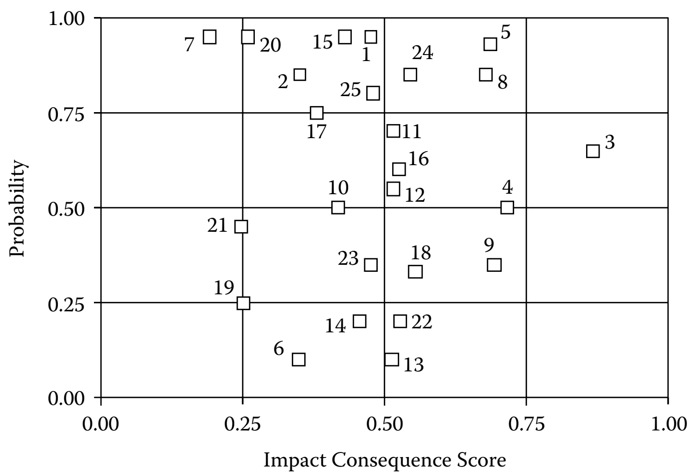

Figure 4.16: A scatter plot of 25 risk events.

is an additive value function. Here, we define the risk score of risk event E by

$$Risk\ Score\,(E) = u_1 Prob\,(E) + u_2 V_{Impact}\,(E) \tag{4.8}$$

where coefficients u_1 and u_2 are non-negative weights that sum to one, the first term is a value function for the event's occurrence probability, and the second term is a value function for the event's overall impact on the project; that is,

$$V_{Impact}\,(E) = w_1 V_{Cost}\,(x_1) + w_2 V_{Sched}\,(x_2) + w_3 V_{TPerf}\,(x_3) + w_4 V_{Prgm}\,(x_4)$$

as defined by Equation 4.5. In Equation 4.8, risk score values will range between zero to one. The higher a risk event's risk score the higher its rank position in the set of identified risk events.

In Equation 4.8, values for the first term derive from a single dimensional value function that represents the probability scale in Table 4.3. Such a value function should be decided and designed by the engineering team in much the same way they are done for the impact (consequence) evaluation criteria. For discussion purposes, a linear relationship between a risk event's occurrence probability and its value function value is assumed. This is shown in Figure 4.17. Nonlinear relationships are certainly possible.

Table 4.12 presents the data for each risk event plotted in Figure 4.16. From left to right, column one is the risk event number, as labeled in Figure 4.16. Column two is the assessment of each risk event's occurrence probability. Column three is each risk event's overall impact score, computed by an application of Equation 4.5. Column four is each risk event's risk score, computed by Equation 4.8. Here, we've assumed a risk event's overall impact score is twice as important as its assessed occurrence probability. This assumption leads to *a form* of the risk score

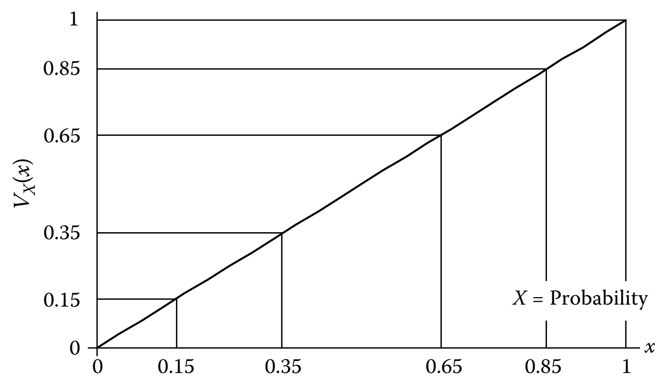

Figure 4.17: A value function for occurrence probability.

TABLE 4.12: Values and Scores for Risks in Figure 4.16

Value Scores Risk Event #	Direct Probability Assessment	Impact Score	Risk Score
1	0.95	0.477	0.635
2	0.85	0.353	0.519
3	0.65	0.867	0.795
4	0.50	0.718	0.645
5	0.93	0.688	0.769
6	0.10	0.349	0.266
7	0.95	0.194	0.446
8	0.85	0.681	0.737
9	0.35	0.695	0.580
10	0.50	0.420	0.447
11	0.70	0.516	0.578
12	0.55	0.517	0.528
13	0.10	0.515	0.376
14	0.20	0.455	0.370
15	0.95	0.432	0.605
16	0.60	0.525	0.550
17	0.75	0.382	0.505
18	0.33	0.555	0.480
19	0.25	0.254	0.252
20	0.95	0.260	0.490
21	0.45	0.248	0.315
22	0.20	0.530	0.420
23	0.35	0.475	0.434
24	0.85	0.545	0.646
25	0.80	0.481	0.587

equation given below.

$$Risk\ Score\,(E) = \frac{1}{3}Prob\,(E) + \frac{2}{3}V_{Impact}\,(E) \tag{4.9}$$

Table 4.13 presents a relative risk ranking based on the value of each risk event's risk score. As mentioned above, the higher a risk event's risk score the higher its rank position relative other identified risks. Thus, risk event 3 is in first rank position. It has the highest risk score in the set shown in Table 4.12. Risk event 5 is in second rank position. It has the second highest risk score in the set shown in Table 4.12, and so forth.

TABLE 4.13: A Relative Ranking of the Risks in Figure 4.16

Risk Ranking	
Risk Event #	**Risk Score**
3	0.795
5	0.769
8	0.737
24	0.646
4	0.645
1	0.635
15	0.605
25	0.587
9	0.580
11	0.578
16	0.550
12	0.528
2	0.519
17	0.505
20	0.490
18	0.480
10	0.447
7	0.446
23	0.434
22	0.420
13	0.376
14	0.370
21	0.315
6	0.266
19	0.252

In Table 4.13, observe the top five ranked risks are risk events 3, 5, 8, 24, and 4. This suggests the project's management should focus further scrutiny on these five risk events to further confirm they indeed merit these critical rank positions. This includes a further look at the basis of assessments behind the value function inputs chosen to characterize each risk event, as these values are used by the risk score equation to generate the above rankings.

Finally, it is best to treat any risk ranking as *indicative or suggestive* of a risk prioritization. Prioritization decisions with respect to where risk mitigation resources should be applied can be guided by this analysis but not solely directed by it. Ranking algorithms are analytical filters that serve as aids to managerial decision-making.

4.3.3 Variations on the Additive Value Model

The above illustrated the use of a simple additive value model as one way to develop a relative rank-order of risks within a set of identified risk events. This section further extends this discussion. Variations on the additive value model are presented, as well as a discussion of other rule types common and new in the engineering community.

Variations on the Additive Value Model

As mentioned previously, risk can be considered a function of its occurrence probability and its impacts to an engineering system project. This is represented by Equation 4.10.

$$Risk = F(Probability,\ Impact) \tag{4.10}$$

What functional form is appropriate for this relationship? The answer is many. This section explores a few of these forms and offers variations on the additive value model. As we'll see, these variations are just some among many that can be designed to reflect the risk attitude of a project team or decision-maker. We will refer to these variations as formulation A, formulation B, and so forth.

Formulation A

A Weighted Linear Combination of Occurrence Probability and Impact

Here, we define the risk score of risk event E by Equation 4.11; that is,

$$Risk\ Score\,(E) = u_1 Prob\,(E) + u_2 V_{Impact}\,(E) \tag{4.11}$$

where coefficients u_1 and u_2 are non-negative weights that sum to one. The first term is a value function for the event's occurrence probability. The second term is a value function for the event's overall impact on the project; that is,

$$V_{Impact}\,(E) = w_1 V_{Cost}\,(x_1) + w_2 V_{Sched}\,(x_2) + w_3 V_{TPerf}\,(x_3) + w_4 V_{Prgm}\,(x_4)$$

as defined by Equation 4.5. In Equation 4.11, risk score values range between zero to one. The higher a risk event's risk score the higher its rank position in the set of identified risk events. Formulation A is the same as the earlier discussion on Equation 4.8.

Formulation B

A "Step-Wise" Linear Combination of Occurrence Probability and Impact

Here, we define the risk score of risk event E by Equation 4.12; that is,

$$Risk\ Score\,(E) = \begin{cases} 1 \quad \text{if } V_{Impact}\,(E) = 1 \\ u_1 Prob\,(E) + u_2 V_{Impact}\,(E) \text{ otherwise} \end{cases} \tag{4.12}$$

where the terms in Equation 4.12 are the same as defined in formulation A, but with the following change. The overall impact score of risk event E, denoted by $V_{Impact}\,(E)$, defaults to one (the maximum score) if *any* of the single dimensional value functions that constitute the terms in $V_{Impact}(E)$ reaches the value of one. If this condition arises, then the overall risk score of event E defaults to a value of one.

The philosophy behind formulation B is as follows. If a risk event E can have a maximum impact (consequence) in *any* of the specified evaluation criteria (i.e., a project's cost, schedule, technical performance, programmatic dimensions) then the overall impact score of E defaults to one; that is,

$$V_{Impact}\,(E) = 1$$

If $V_{Impact}(E) = 1$ then, according to formulation B, the risk score of event E defaults to its maximum value, which is one. Thus, if a risk event's occurrence would cause a level 5 impact on any of the project's consequence dimensions then such an effect would not be "diluted" by a weighted average rule — as in formulation A. In formulation B, risk events with a level 5 impact will always be visible to the project's management regardless of their occurrence probabilities or the importance weights of the impact (consequence) evaluation criteria.

A modification to formulation B might be as follows. The project team defines a threshold level for all values produced by the value functions used to evaluate a risk event's impact. If, for some risk event E, *each value function produces a value at or above* this threshold, then the risk score of E is equal to the maximum of that set of value function values; otherwise, the risk score of risk event E is computed by Equation 4.11. An illustration of this algorithm is shown in Figure 4.18.

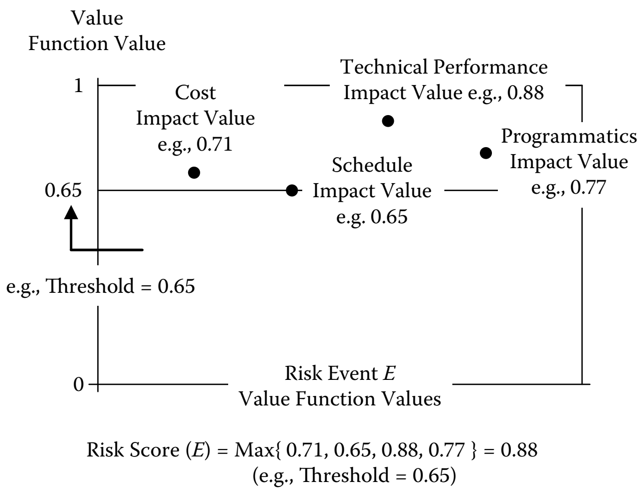

Figure 4.18: A threshold level case.

Formulation C

Maximum "Max" Average

Here, we introduce a new measure called the "max" average.* Its application as an algorithm for rank-ordering risks in a set of identified risk events will be shown. First, the definition is presented.

Definition 4.1 The max average of $x_1, x_2, x_3, \ldots, x_n$ where $0 \leq x_i \leq 1$ for all $i = 1, 2, 3, \ldots, n$ is

$$Max\ Ave = \lambda m + (1 - \lambda) Average\{x_1, x_2, x_3, \ldots, x_n\} \tag{4.13}$$

where $m = Max\{x_1, x_2, x_3, \ldots, x_n\}$ and λ is a weighting function given by Equation 4.14.

The weighting function λ in Equation 4.14 is shown in Figure 4.19. This is one of many possible forms of a weight function. Another form for λ is shown in

*The max average was created by Dr. Bruce W. Lamar (MITRE, 2005) and published by The MITRE Corporation in the paper *Min-Additive Utility Functions*, MPO80070-1, April 2008, © 2008, All Rights Reserved.

The weighting function in equation 4.14 is a form of the sigmoid function. The sigmoid is a mathematical function that can produce curves with an "S-like" shape.

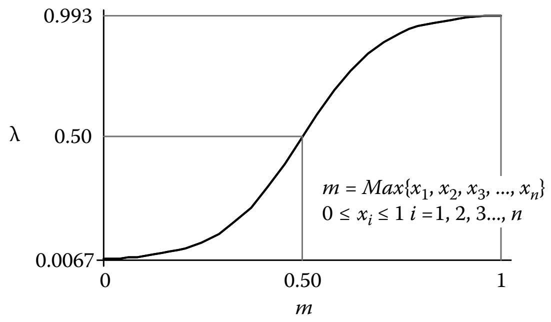

Figure 4.19: A max average weighting function λ.

Figure 4.53 as part of the discussion in Section 4.6.2.

$$0 < \lambda = 1 - \frac{1}{1 + e^{10(m-1/2)}} = \frac{e^{10(m-1/2)}}{1 + e^{10(m-1/2)}} < 1 \qquad (4.14)$$

In Equation 4.14, if $m > 0.50$ then the max average of $x_1, x_2, x_3, \ldots, x_n$ is weighted toward the maximum of $\{x_1, x_2, x_3, \ldots, x_n\}$. If $m < 0.50$, then the max average of $x_1, x_2, x_3, \ldots, x_n$ is weighted toward the arithmetic average of $x_1, x_2, x_3, \ldots, x_n$. If $m = 0.50$, then the max average of $x_1, x_2, x_3, \ldots, x_n$ is weighted equally ($\lambda = 0.50$) between the maximum of $x_1, x_2, x_3, \ldots, x_n$ and the arithmetic average of $x_1, x_2, x_3, \ldots, x_n$. Next, the max average will be applied to the problem of rank-ordering risks in a set of identified events.

One way to apply the max average rule is to define $V_{Impact}(E)$ as follows:

$$\begin{aligned} V_{Impact}(E) = {} & \lambda Max\{v_{Cost}, v_{Sched}, v_{TPerf}, v_{Prgm}\} \\ & + (1-\lambda) Weighted\ Average\{w_1 v_{Cost}, w_2 v_{Sched}, w_3 v_{TPerf}, w_4 v_{Prgm}\} \end{aligned} \qquad (4.15)$$

where $v_{Cost} = V_{Cost}(x_1)$, $v_{Sched} = V_{Sched}(x_2)$, $v_{TPerf} = V_{TPerf}(x_3)$, and $v_{Prgm} = V_{Prgm}(x_4)$ are defined by Equation 4.5. Here, w_i (for $i = 1, \ldots, 4$) are non-negative importance weights with values that range between zero and one and where $w_1 + w_2 + w_3 + w_4 = 1$.

The second term in Equation 4.15 is computed as follows:

$$\begin{aligned} & Weighted\ Average\{w_1 v_{Cost}, w_2 v_{Sched}, w_3 v_{TPerf}, w_4 v_{Prgm}\} \\ & = w_1 v_{Cost} + w_2 v_{Sched} + w_3 v_{TPerf} + w_4 v_{Prgm} \end{aligned} \qquad (4.16)$$

Using Equation 4.15 for $V_{Impact}(E)$, formulation A could then be used to compute the risk score of event E; that is,

$$Risk\ Score\,(E) = u_1 Prob\,(E) + u_2 V_{Impact}(E) \tag{4.17}$$

Example 4.1 From Case Discussion 4.1, compute the risk score of risk event A using Equation 4.17 and the max average rule given by Equation 4.15. Here, assume the overall impact score of risk event A is twice as important as its assessed occurrence probability.

Solution From Case Discussion 4.1, Equation 4.8, and the assumption that risk event A's overall impact score is twice as important as its assessed occurrence probability we have the following:

$$Risk\ Score(A) = \frac{1}{3} Prob\,(A) + \frac{2}{3} V_{Impact}\,(A) \tag{4.18}$$

Next, we'll use the max average rule to compute $V_{Impact}(A)$. From Case Discussion 4.1, Table 4.11, and Equation 4.15 we have the following:

$$v_{Cost} = 0.842, \quad v_{Sched} = 0.604, \quad v_{TPerf} = 0.60, \quad \text{and} \quad v_{Prgm} = 0.79$$

From the above, it follows that

$$m = Max\{v_{Cost}, v_{Sched}, v_{TPerf}, v_{Prgm}\} = Max\{0.842, 0.604, 0.60, 0.79\} = 0.842$$

Next, we compute

$$\begin{aligned} &Weighted\ Average\{w_1 v_{Cost}, w_2 v_{Sched}, w_3 v_{TPerf}, w_4 v_{Prgm}\} \\ &\quad = w_1 v_{Cost} + w_2 v_{Sched} + w_3 v_{TPerf} + w_4 v_{Prgm} \end{aligned} \tag{4.19}$$

where, from Case Discussion 4.1, the weights were 1/4, 1/8, 1/2, 1/8. Hence

$$\begin{aligned} &Weighted\ Average\left\{\frac{1}{4} v_{Cost}, \frac{1}{8} v_{Sched}, \frac{1}{2} v_{TPerf}, \frac{1}{8} v_{Prgm}\right\} \\ &\quad = \frac{1}{4} v_{Cost} + \frac{1}{8} v_{Sched} + \frac{1}{2} v_{TPerf} + \frac{1}{8} v_{Prgm} = 0.685 \end{aligned} \tag{4.20}$$

From Equation 4.15 we have

$$V_{Impact}(A) = \lambda(0.842) + (1 - \lambda)(0.685)$$

where

$$\lambda = 1 - \frac{1}{1 + e^{10(m-1/2)}} = 1 - \frac{1}{1 + e^{10(0.842-1/2)}} = 0.968$$

It follows that

$$V_{Impact}(A) = 0.968(0.842) + (1 - 0.968)(0.685) = 0.837$$

from which

$$Risk\ Score(A) = \frac{1}{3}Prob(A) + \frac{2}{3}V_{Impact}(A) \quad (4.21)$$

$$Risk\ Score(A) = \frac{1}{3}(0.95) + \frac{2}{3}(0.837) = 0.875 \quad (4.22)$$

Figure 4.20 shows a plot of risk event A. Two points are shown. The left-most point is a plot of risk event A if the event's overall impact score is computed by a strict weighted average rule (refer to Equation 4.7). The right-most point is a plot of risk event A if the event's overall impact score is computed using the max average rule (refer to the use of Equation 4.15 in example 4.1). Why is there such a difference? When do you choose one rule over another?

The answer to the first question can be seen in the technical differences between Equations 4.7 and 4.15. In Equation 4.7, a risk event's overall impact score is a weighted average of its individual impact scores (i.e., the value function values) across the evaluation criteria. Equation 4.15 uses this same weighted average in

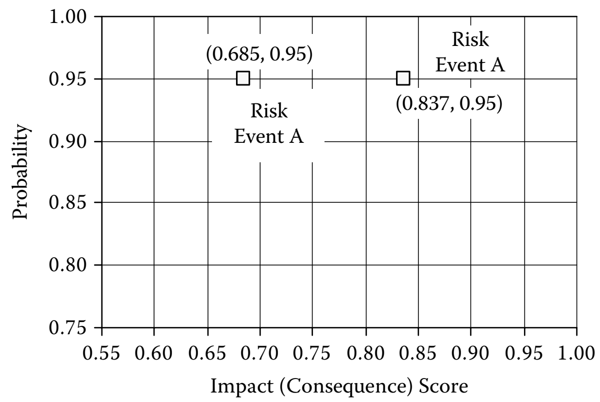

Figure 4.20: Example 4-1: a plot of risk event A: two scoring rules.

its second term; however, the first term of Equation 4.15 takes the maximum value of the risk event's individual impact scores across the evaluation criteria.

In Example 4.1, the maximum value received almost 97% of the importance weight while the weighted average received approximately 3% of the importance weight. In this case, the maximum of the individual impact scores dominated the overall impact score.

The answer to the second question is driven by the risk attitude or impact sensitivity of the project team. The max average rule "values" the maximum m more than the weighted average when m is greater than 0.50. The max average rule "values" the average more than the maximum when m is less than 0.50. The max average rule "equally values" the maximum m and the average when m is equal to 0.50, each receiving a weight equal to one-half. These characteristics can be seen by a close examination of Figure 4.19, a graph of Equation 4.14.

Formulation D

Product Rule

The "product rule" is a popular formulation in the general risk management community. The product rule defines the risk score of risk event E as the product of the event's occurrence probability and its impact (consequence). A traditional form of the product rule is given by Equation 4.23.

$$Risk\ Score(E) = Prob(E) \cdot V_{Impact}(E) \tag{4.23}$$

Here, the first term is a value function for the event's occurrence probability.* The second term is a value function for the event's overall impact on the project, as defined by Equation 4.5.

From a statistical perspective, the product rule generates a measure known as an expected value [5]. In this context, *Risk Score*(E) could be interpreted as the expected impact of risk event E.

The rule given by Equation 4.23 produces meaningful results *only* when its terms are *defined along cardinal scales*. For reasons discussed in Chapter 3 and in section 4.3.1, Equation 4.23 should not be used when either *Prob(E)* or $V_{Impact}(E)$ are ordinal valued.

*Refer to the discussion on equation 4.8 and Figure 4.17 for a further discussion of this value function.

A shortcoming with the product rule is the inability to weight the importance of $Prob(E)$ versus $V_{Impact}(E)$. However, importance weights for the evaluation criteria that constitute $V_{Impact}(E)$ can be weighted. Here, Equation 4.23 could be written as follows:*

$$\begin{aligned} Risk\ Score(E) &= Prob(E) \cdot V_{Impact}(E) \\ &= Prob(E) \cdot (w_1 v_{Cost} + w_2 v_{Sched} + w_3 v_{TPerf} + w_4 v_{Prgm}) \end{aligned} \tag{4.24}$$

where $v_{Cost} = V_{Cost}(x_1)$, $v_{Sched} = V_{Sched}(x_2)$, $v_{TPerf} = V_{TPerf}(x_3)$, and $v_{Prgm} = V_{Prgm}(x_4)$ are defined by Equation 4.5. Here, w_i (for $i = 1, \ldots, 4$) are non-negative importance weights with values that range between zero and one and where $w_1 + w_2 + w_3 + w_4 = 1$.

Related to the above, the product rule is structured in a way where an event's occurrence probability can significantly discount its overall potential impact (consequence) to a project. The preceding formulations had an event's occurrence probability as additive to impact (consequence) and not multiplicative. In some circumstances, Equation 4.23 can give a false sense of comfort. For example, a high-impact risk event can appear less threatening to a project if its occurrence probability is determined to be low. Allowing occurrence probability this much "influence" on an event's risk score may not be a good idea, since a subjective assessment of an event's occurrence probability can be off by a wide margin and is often made without strong defensible arguments. In practice, projects tend to be more sensitive to a risk event's impact than its probability. As such, a high-consequence risk event should never lose visibility or be mistakenly downplayed, because its occurrence probability is merely considered low. Program managers and decision-makers should always be presented with both values.

There are other related issues associated with this rule. The product rule can produce risk scores that are the same, or close in value, for two very different risk events. Seen in Figure 4.21, a risk event with a low-impact and a high-occurrence probability can produce the same risk score as an event with a high-impact and a low-occurrence probability. In these circumstances, the individual values for $Prob(E)$ and $V_{Impact}(E)$ should be flagged so program managers and decision-makers can assess where tradeoffs may best be made.

*Formulas for $V_{Impact}(E)$ developed in formulations B and C could also be used in the "product-rule" formulation for risk score.

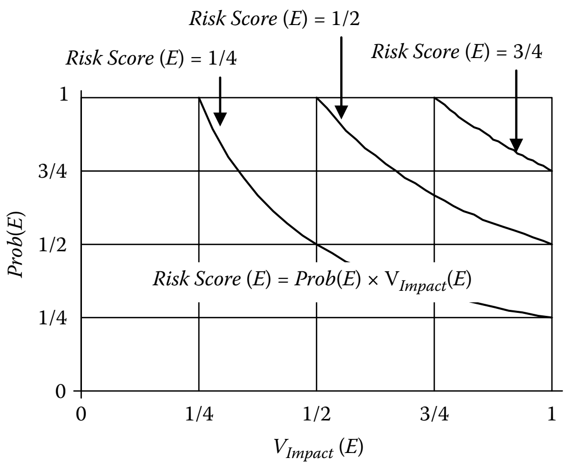

Figure 4.21: Curves of constant risk score.

Example 4.2 From Case Discussion 4.1, compute the risk score of risk event A using the product rule with

a) $V_{Impact}(E)$ derived from Equation 4.6 and Equation 4.7

b) $V_{Impact}(E)$ derived from Equation 4.15 in example 4.1

Solution From Case Discussion 4.1, we have the following:

$B = \{$The new database for the communication system will not be fully tested for compatibility with the existing subsystem databases when version 1.0 of the data management architecture is released$\}$

$A = \{$Inadequate synchronization of the communication system's new database with the existing subsystem databases$\}$

It was also assessed that the occurrence probability of risk event A was

$$0 < P(A|B) = \alpha = 0.95 < 1$$

Using the product rule, we would write:

$$Risk\ Score(A) = 0.95 \cdot V_{Impact}(E) \tag{4.25}$$

(a) From Equation 4.6, we determined that

$$V_{Impact}(A) = \frac{1}{4}V_{Cost}(x_1) + \frac{1}{8}V_{Sched}(x_2) + \frac{1}{2}V_{TPerf}(x_3) + \frac{1}{8}V_{Prgm}(x_4)$$

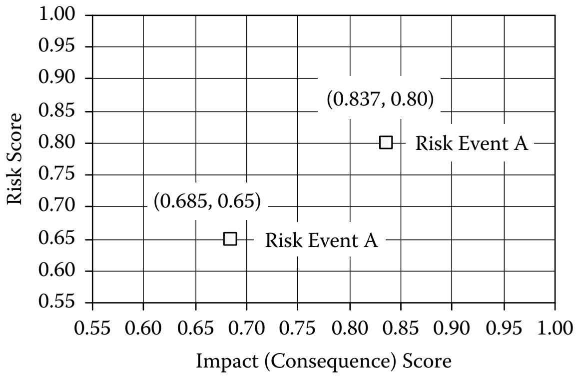

Figure 4.22: A plot of risk score versus impact: Example 4.2.

which, from Case Discussion 4.1 (Equation 4.7) is

$$V_{Impact}(A) = \frac{1}{4}(0.842) + \frac{1}{8}(0.604) + \frac{1}{2}(0.60) + \frac{1}{8}(0.79) = 0.685 \qquad (4.26)$$

It follows that

$$Risk\ Score(A) = 0.95 \cdot (0.685) = 0.65 \qquad (4.27)$$

(b) From Equation 4.15 and Example 4.1 we have

$$V_{Impact}(A) = 0.837 \qquad (4.28)$$

It follows that

$$Risk\ Score(A) = 0.95 \cdot (0.837) = 0.80 \qquad (4.29)$$

Figure 4.22 presents a plot of these two values for risk score, as a function of risk event A's impact.

$$Risk\ Score(A) = Prob(A) \cdot V_{Impact}(A)$$

4.3.4 Incorporating Uncertainty

. . . The only certainty is uncertainty.

— Pliny the Elder (Gaius Plinius Secundus)

This section illustrates an application of the power-additive utility function as a way to capture uncertainty, and the risk attitude of the decision-maker, in the analysis of one or more risk events. The power-additive utility function was introduced in Chapter 3. It is a function that takes values from a multiattribute value function and maps them into a corresponding set of utilities in accordance with the risk attitude of the decision-maker. The power-additive utility function covers a wide span of possible risk attitudes, as shown in Figure 3.24. In this section, we limit our focus to the situation where the utilities are monotonically increasing.

Concept Review (from Chapter 3)

Recall the following from Chapter 3, Definition 3.12. If utilities are *monotonically increasing* over the values of the additive value function $V_Y(y)$, then the power-additive utility function is given by

$$U(v) = \begin{cases} K(1 - e^{-(V_Y(y)/\rho_m)}) & \text{if } \rho_m \neq \infty \\ V_Y(y) & \text{if } \rho_m = \infty \end{cases} \tag{4.30}$$

where $K = 1/(1 - e^{-1/\rho_m})$, $v = V_Y(y) = \sum_{i=1}^{n} w_i V_{X_i}(x_i)$, ρ_m is the multiattribute risk tolerance, and $V_Y(y)$ is the additive value function given in Definition 3.6.

From Chapter 3, recall that expected utilities provide measures with which to rank uncertain alternatives, from most- to least-preferred. In a risk event context, we will compute the expected utilities of their values as a way to rank them, from most- to least-critical, when uncertainties are present in the characteristics of these events.

From Theorem 3.4, if utilities are *monotonically increasing* over the attributes of the additive value function $V_Y(y)$, then the expected utility $E(U(v))$ is given below.

$$E(U(v)) = \begin{cases} K(1 - E(e^{-(V_Y(y)/\rho_m)})) & \text{if } \rho_m \neq \infty \\ E(V_Y(y)) & \text{if } \rho_m = \infty \end{cases} \tag{4.31}$$

For the case where $\rho \neq \infty$, the term $E(e^{-(V_Y(y)/\rho_m)})$ can be written as follows:

$$E(e^{-(V_Y(y)/\rho_m)}) = E(e^{-(w_1 V_{X_1}(x_1) + w_2 V_{X_2}(x_2) \ldots w_n V_{X_n}(x_n))/\rho_m})$$
$$E(e^{-(V_Y(y)/\rho_m)}) = E(e^{-(w_1 V_{X_1}(x_1))/\rho_m}) E(e^{-(w_2 V_{X_2}(x_2))/\rho_m}) \ldots E(e^{-(w_n V_{X_n}(x_n))/\rho_m})$$

where the X_i's are independent random variables and where

$$E(e^{-(w_i V_{X_i}(x_i))/\rho_m}) = \begin{cases} \sum_{x_i} p_{X_i}(x_i)\, e^{-(w_i V_{X_i}(x_i))/\rho_m} & \text{if } X_i \text{ is discrete} \\ \int_{-\infty}^{\infty} e^{-(w_i V_{X_i}(x_i))/\rho_m} f_{X_i}(x_i)\, dx_i & \text{if } X_i \text{ is continuous} \end{cases} \tag{4.32}$$

In the above, $p_{X_i}(x_i)$ is the probability the uncertain outcome X_i takes the score x_i if X_i is a discrete random variable and $f_{X_i}(x_i)$ is the probability density function of X_i if X_i is a continuous random variable.

Case Discussion 4.1a: An Application Illustration

The following is an extension of Case Discussion 4.1. It shows how to compute the expected utility of the value of a risk event, for a given risk attitude, in the presence of uncertainty in the parameters that characterize the event.

Although the approach in Case Discussion 4.1a is illustrated for one risk event, it can be applied to each risk event in a set of identified events; that is, the expected utility of the value of each risk can be computed as (1) a function of the uncertainties that characterize each event and (2) the risk attitude of the program manager or decision-maker. From this, a most- to least-critical ranking of each risk event can be determined from these expected utility measures. Higher criticality events have higher expected utilities, and so forth.

Computing the Expected Utility

From the value functions given in Table 4.14, we can write the following additive value function.

$$V_Y(y) = u_1 V_Z(z) + u_2(w_1 V_{X_1}(x_1) + w_2 V_{X_2}(x_2) + w_3 V_{X_3}(x_3) + w_4 V_{X_4}(x_4))$$

Here, we assume the conditions defined in Chapter 3 for an additive value function hold. Observe the preceding equation is an equivalent formulation to Equation 4.8, called *Risk Score*. Here, the first term is the value function for the occurrence probability of the risk event (refer to Figure 4.17). The remaining terms are equivalent to the terms that make up the value function for the risk event's overall impact, defined by Equation 4.5; that is, X_1 denotes the criterion Cost Impact; X_2 denotes the criterion Schedule Impact, X_3 denotes the criterion Technical Performance Impact, and X_4 denotes the criterion Programmatic Impact.

Applying the weights from Case Discussion 4.1, and Equation 4.9, we have the following:

$$V_Y(y) = \frac{1}{3} V_Z(z) + \frac{2}{3}\left(\frac{1}{4} V_{X_1}(x_1) + \frac{1}{8} V_{X_2}(x_2) + \frac{1}{2} V_{X_3}(x_3) + \frac{1}{8} V_{X_4}(x_4)\right)$$

TABLE 4.14: Uncertainty Assessments and Scores from Case Discussion 4.1

Risk Event A	**Basis of Assessment**
Uncertainty Assessments	Risk Statement *Inadequate synchronization of the communication system's new database with the existing subsystem databases, because the new database for the communication system will not be fully tested for compatibility with the existing subsystem databases when version 1.0 of the data management architecture is released.*
Criterion Z Occurrence Probability	This risk event is assessed to have between an 85% and a 95% chance of occurrence. Value Function: From Figure 4.17, $V_Z(z) = z$ *Note: Suppose the uncertainty is uniformly distributed.*
Criterion X_1 Cost Impact	This risk event, if it occurs, is estimated by the engineering team to cause a 10% to 15% increase in the project's current budget. The estimate is based on a careful assessment of the ripple effects across the project's cost categories for interoperability fixes to the databases, the supporting software, and the extent that retesting is needed. Value Function: From Equation 4.2, $V_{X_1}(x_1) = 1.096(1 - e^{-x_1/8.2})$ *Note: Suppose the uncertainty is uniformly distributed.*
Criterion X_2 Schedule Impact	This risk event, if it occurs, is estimated by the engineering team to cause a 3-month to 6-month increase in the project's current schedule. The estimate is based on a careful assessment of the ripple effects across the project's integrated master schedule for interoperability-related fixes to the databases, the supporting software, and the extent that retesting is needed. Value Function: From Equation 4.3, $V_{X_2}(x_2) = 1.018(1 - e^{-x_2/4.44})$ *Note: Suppose the uncertainty is uniformly distributed.*

TABLE 4.14: Uncertainty Assessments and Scores from Case Discussion 4.1 (*Continued*)

Risk Event A	Basis of Assessment
Criterion X_3 Technical Performance Impact	This risk event, if it occurs, is assessed by the engineering team as one that will impact the system's operational capabilities to the extent that technical performance is marginally below minimum acceptable levels, depending on the location and extent of interoperability shortfalls. Given this, suppose the engineering team assessed a 75% chance this risk event would have a Level 4 technical performance impact and a 25% chance it would have a Level 3 impact. Value Function: From Figure 4.12, $V_{X_3}(4) = 9/15$ and $V_{X_3}(3) = 5/15$
Criterion X_4 Programmatic Impact	This risk event, if it occurs, is assessed by the engineering team as one that will impact programmatic efforts to the extent that one or more stated objectives for technical or programmatic work products (e.g., various specifications or activities) is marginally below minimum acceptable levels. Given this, suppose the engineering team assessed a 60% chance this risk event would have a Level 4 technical performance impact and a 40% chance it would have a Level 3 impact. Value Function: From Figure 4.12, $V_{X_4}(4) = 15/19$ and $V_{X_4}(3) = 9/19$

which is equal to

$$V_Y(y) = \frac{1}{3}V_Z(z) + \frac{1}{6}V_{X_1}(x_1) + \frac{1}{12}V_{X_2}(x_2) + \frac{1}{3}V_{X_3}(x_3) + \frac{1}{12}V_{X_4}(x_4) \quad (4.33)$$

Suppose decision-makers reviewed the graphs in Figure 3.24 (Chapter 3) and assessed their multiattribute risk tolerance as represented by the curve with $\rho_m = 1$. So, their risk preference structure reflects a monotonically increasing, slightly risk-averse attitude over increasing values of the value function in Equation 4.33. From this, and the information in Table 4.14, we can now compute the expected utility of the value of the risk event defined in Case Discussion 4.1.

From Theorem 3.4 we have

$$E(U(v)) = K(1 - E(e^{-(V_Y(y)/\rho_m)})) \tag{4.34}$$

where $K = 1/(1 - e^{-1/\rho_m})$ and $v = V_Y(y)$ as given by Equation 4.33. It follows that with $\rho_m = 1$ Equation 4.34 becomes

$$E(U(v)) = 1.582(1 - E(e^{-V_Y(y)})) \tag{4.35}$$

In Equation 4.35, we have

$$v = V_Y(y) \tag{4.36}$$

$$= \frac{1}{3}V_Z(z) + \frac{1}{6}V_{X_1}(x_1) + \frac{1}{12}V_{X_2}(x_2) + \frac{1}{3}V_{X_3}(x_3) + \frac{1}{12}V_{X_4}(x_4) \tag{4.37}$$

where

$$V_Z(z) = z \tag{4.38}$$

$$V_{X_1}(x_1) = 1.096(1 - e^{-x_1/8.2}) \tag{4.39}$$

$$V_{X_2}(x_2) = 1.018(1 - e^{-x_2/4.44}) \tag{4.40}$$

and $V_{X_3}(x_3)$ and $V_{X_4}(x_4)$ are given by the value functions in Figure 4.12. Next, we will look at computing the term $E(e^{-V_Y(y)})$ in Equation 4.35. Here,

$$E(e^{-V_Y(y)}) = E(e^{-(\frac{1}{3}V_Z(z)+\frac{1}{6}V_{X_1}(x_1)+\frac{1}{12}V_{X_2}(x_2)+\frac{1}{3}V_{X_3}(x_3)+\frac{1}{12}V_{X_4}(x_4))})$$

If we assume Z, X_1, X_2, X_3, and X_4 are independent random variables* then

$$E(e^{-V_Y(y)}) = E(e^{-\frac{1}{3}V_Z(z)})E(e^{-\frac{1}{6}V_{X_1}(x_1)})E(e^{-\frac{1}{12}V_{X_2}(x_2)})E(e^{-\frac{1}{3}V_{X_3}(x_3)})E(e^{-\frac{1}{12}V_{X_4}(x_4)}) \tag{4.41}$$

*This assumption will be discussed further at the end of this section.

where

$$E(e^{-\frac{1}{3}V_Z(z)}) = \int_{-\infty}^{\infty} e^{-\frac{1}{3}V_Z(z)} f_Z(z)\, dz \tag{4.42}$$

$$E(e^{-\frac{1}{6}V_{X_1}(x_1)}) = \int_{-\infty}^{\infty} e^{-\frac{1}{6}V_{X_1}(x_1)} f_{X_1}(x_1)\, dx_1 \tag{4.43}$$

$$E(e^{-\frac{1}{12}V_{X_2}(x_2)}) = \int_{-\infty}^{\infty} e^{-\frac{1}{12}V_{X_2}(x_2)} f_{X_2}(x_2)\, dx_2 \tag{4.44}$$

$$E(e^{-\frac{1}{3}V_{X_3}(x_3)}) = \sum_{x_3} p_{X_3}(x_3)\, e^{-\frac{1}{3}V_{X_3}(x_3)} \tag{4.45}$$

$$E(e^{-\frac{1}{12}V_{X_4}(x_4)}) = \sum_{x_4} p_{X_4}(x_4)\, e^{-\frac{1}{12}V_{X_4}(x_4)} \tag{4.46}$$

and $f_Z(z)$, $f_{X_i}(x_i)$, and $p_{X_i}(x_i)$ are probability distributions for Z and X_i which, in this case discussion, are stated in Table 4.14. From Table 4.14, the above can be computed, as given below.

$$E(e^{-\frac{1}{3}V_Z(z)}) = \int_{0.85}^{0.95} e^{-\frac{1}{3}z} \frac{1}{0.95-0.85}\, dz = 0.740853 \tag{4.47}$$

$$E(e^{-\frac{1}{6}V_{X_1}(x_1)}) = \int_{10}^{15} e^{-\frac{1}{6}(1.096(1-e^{-x_1/8.2}))} \frac{1}{15-10}\, dx_1 = 0.867408 \tag{4.48}$$

$$E(e^{-\frac{1}{12}V_{X_2}(x_2)}) = \int_{3}^{6} e^{-\frac{1}{12}(1.018(1-e^{-x_2/4.44}))} \frac{1}{6-3}\, dx_2 = 0.947966 \tag{4.49}$$

$$\begin{aligned} E(e^{-\frac{1}{3}V_{X_3}(x_3)}) &= \sum_{x_3} p_{X_3}(x_3)\, e^{-\frac{1}{3}V_{X_3}(x_3)} \\ &= 0.75 e^{-\frac{1}{3}\frac{9}{15}} + 0.25 e^{-\frac{1}{3}\frac{5}{15}} = 0.837758 \end{aligned} \tag{4.50}$$

$$\begin{aligned} E(e^{-\frac{1}{12}V_{X_4}(x_4)}) &= \sum_{x_4} p_{X_4}(x_4)\, e^{-\frac{1}{12}V_{X_4}(x_4)} \\ &= 0.60 e^{-\frac{1}{12}\frac{15}{19}} + 0.40 e^{-\frac{1}{12}\frac{9}{19}} = 0.946315 \end{aligned} \tag{4.51}$$

Entering these values into Equation 4.41 we have

$$E(e^{-V_Y(y)}) = (0.740853)(0.867408)(0.947966)(0.837758)(0.946315) = 0.48295 \quad (4.52)$$

Substituting this value for $E(e^{-V_Y(y)})$ into Equation 4.35 we have

$$E(U(v)) = 1.582(1 - 0.48295) = 0.817961 \quad (4.53)$$

Computing the Expected Value

Next, we proceed to compute the expected value of the risk event's value. Here, we need to determine $E(v)$ where,

$$\begin{aligned} E(v) &= E(V_Y(y)) \\ &= E\left(\frac{1}{3}V_Z(z) + \frac{1}{6}V_{X_1}(x_1) + \frac{1}{12}V_{X_2}(x_2) + \frac{1}{3}V_{X_3}(x_3) + \frac{1}{12}V_{X_4}(x_4)\right) \\ &= \frac{1}{3}E(V_Z(z)) + \frac{1}{6}E(V_{X_1}(x_1)) + \frac{1}{12}E(V_{X_2}(x_2)) \\ &\qquad + \frac{1}{3}E(V_{X_3}(x_3)) + \frac{1}{12}E(V_{X_4}(x_4)) \end{aligned}$$

The terms in the above expression are determined, in this case, as follows:

$$E(V_Z(z)) = \int_{-\infty}^{\infty} V_Z(z)\, f_Z(z)dz = \int_{0.85}^{0.95} z\, \frac{1}{0.95 - 0.85}dz = 0.90$$

$$\begin{aligned} E(V_{X_1}(x_1)) &= \int_{-\infty}^{\infty} V_{X_1}(x_1)\, f_{X_1}(x_1)dx_1 = \int_{10}^{15} 1.096(1 - e^{-x_1/8.2})\, \frac{1}{15 - 10}dx_1 \\ &= 0.853627 \end{aligned}$$

$$\begin{aligned} E(V_{X_2}(x_2)) &= \int_{-\infty}^{\infty} V_{X_2}(x_2)\, f_{X_2}(x_2)dx_2 = \int_{3}^{6} 1.018(1 - e^{-x_2/4.44})\, \frac{1}{6 - 3}dx_2 \\ &= 0.641457 \end{aligned}$$

$$E(V_{X_3}(x_3)) = \sum_{x_3} p_{X_3}(x_3) V_{X_3}(x_3) = 0.75 \left(\frac{9}{15}\right) + 0.25 \left(\frac{5}{15}\right)$$

$$= 0.533333$$

$$E(V_{X_4}(x_4)) = \sum_{x_4} p_{X_4}(x_4) V_{X_4}(x_4) = 0.60 \left(\frac{15}{19}\right) + 0.40 \left(\frac{9}{19}\right) = 0.663158$$

Substituting these values into the above expression for $E(v)$ we have

$$E(v) = \frac{1}{3}(0.90) + \frac{1}{6}(0.853627) + \frac{1}{12}(0.641457) + \frac{1}{3}(0.533333) + \frac{1}{12}(0.663158) = 0.728767 \quad (4.54)$$

Some Observations

This case discussion looked at how to incorporate uncertainty in the parameters that characterized one risk event. Two measures were computed — the expected utility $E(U(v))$ of the value of the risk event and the expected value of its value, denoted by $E(v)$. Here, the value of the risk event was measured by the value function we called *Risk Score*, as given by Equation 4.8.

In the preceding case discussion, the multiattribute risk tolerance was set equal to one ($\rho_m = 1$). This means the program manager or decision-maker had a slight degree of risk averseness over the values of the value function. A graph of this utility function is presented in Figure 4.23.

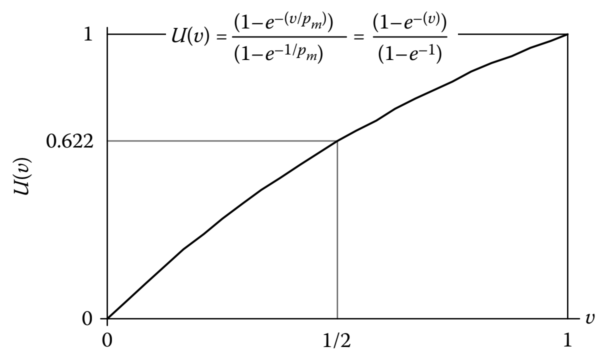

Figure 4.23: Utility function for Case Discussion 4.1a.

From the above, we determined that $E(U(v)) = 0.817961$ and $E(v) = 0.728767$. Recall from Chapter 3 that, in the case of risk averseness, the utility of the expected value should be larger than the expected utility. This can be seen here as well. In this case, we have

$$U(E(v)) = U(0.728767) = 0.818667 > E(U(v)) = 0.817961$$

Next, it might be asked: *What is the value of v associated with the expected utility*? This would be a measure known as the *certainty equivalent value* of the risk event, in this case (refer to Chapter 3). To determine the certainty equivalent value we solve the expression below for v_{CE}.

$$E(U(v)) = 0.817961 = \frac{(1 - e^{-(v_{CE})})}{(1 - e^{-1})} = U(v_{CE})$$

With a little algebra, it can be shown that $v_{CE} = 0.727842$. So, this is the value of the value function that produces the expected utility of the value of the risk event. Notice v_{CE} is slightly less than $E(v)$, as expected. A graph of these observations is shown in Figure 4.24. Figure 4.24 "narrows-in" on the utility function in Figure 4.23 for the region $0.725 \leq v \leq 0.73$. Notice how linear the function looks in this very tight interval.

Finally, let's take another look at Case Discussion 4.1 as originally presented without considering uncertainty or risk preferences (i.e., risk attitudes). Suppose we use formulation A to measure the risk score of risk event A, with weights

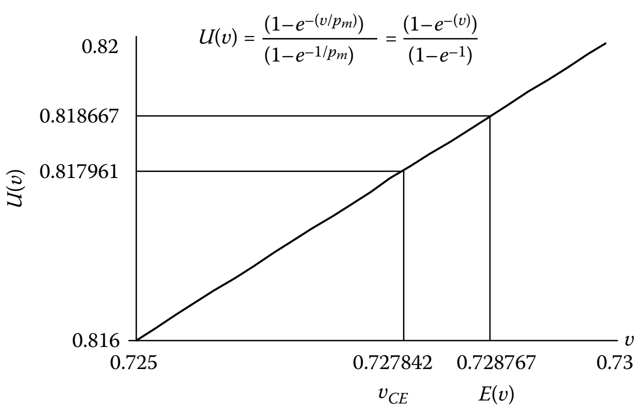

Figure 4.24: Some utility function values for case discussion 4.1a.

shown in Equation 4.55.

$$Risk\ Score\ (A) = \frac{1}{3}Prob(A) + \frac{2}{3}V_{Impact}(A) \qquad (4.55)$$

From the results of Case Discussion 4.1, we can compute the following:

$$Risk\ Score\ (A) = \frac{1}{3}(0.95) + \frac{2}{3}(0.685) = 0.77333 \qquad (4.56)$$

Notice how this value is greater than $E(v)$. The reason is that $E(v)$ incorporates uncertainties in the values for the parameters that characterize the risk event. These uncertainties were given in Table 4.14, where there were enough "lower possible" scores, in this case, to drive $E(v)$ below the value of risk score, as computed by Equation 4.56.

Some Words on Probabilistic Independence

In Case Discussion 4.1a, a key assumption was that Z, X_1, X_2, X_3, and X_4 were independent random variables. There is a practical reason for making this assumption. Without it, Equation 4.41 could not be written as shown.

However, there are occasions when random variables, such as these, are not independent. For example, the uncertainties in the levels (or scores) for Cost (X_1) and Schedule (X_2) can sometimes vary together, not independently. When this occurs, expected utility computations can be in error, if independence assumptions were wrongly made. The error worsens with greater degrees of risk averseness. The error lessens with fewer degrees of risk averseness and lessens substantially when the utility function approaches a straight line, otherwise known as the risk neutral condition. The error goes to zero when the utility function is linear since, in this situation, the expected utility is exactly equal to the expected value. From probability theory [5], it can be shown that expected value computations are not subject to independence or dependence considerations.

When probabilistic independence can't be assumed or demonstrated, one approach is to redefine the dependent random variables in a way that they will exhibit independence from the rest of the random variables in the set. For example, in Case Discussion 4.1a, if Cost (X_1) and Schedule (X_2) were not independent then one could do the following. Redefine the value function for Cost such that it captures the joint interactions between cost and schedule. In this way, the new value